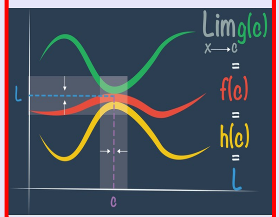
Calculus I

Lecture 20



Feb 19-8:47 AM

Class Quiz 17

Given $f(x) = x^3 - 12x$

$f(2) = 2^3 - 12(2) = 8 - 24 = -16$

$f(-2) = (-2)^3 - 12(-2) = -8 + 24 = 16$

Find all points on the graph of $f(x)$ where

1) $f'(x) = 0$

$f'(x) = 3x^2 - 12$

$f'(x) = 0$

$3x^2 - 12 = 0 \rightarrow x^2 - 4 = 0$

$x = \pm 2$

$(2, -16), (-2, 16)$

2) $f''(x) = 0$

$f''(x) = 6x$

$f''(x) = 0$

$6x = 0 \rightarrow x = 0$

$(0, 0)$

3) Show $f(x)$ is an odd function.

$f(-x) = (-x)^3 - 12(-x) = -x^3 + 12x = -(x^3 - 12x) = -f(x)$

So $f(-x) = -f(x)$ $\therefore$ $f(x)$ is an odd function.

Graph is symmetric w/t origin.

Apr 30-8:12 AM

$f'(x) = 3x^2 - 12$

$= 3(x^2 - 4)$

$= 3(x+2)(x-2)$

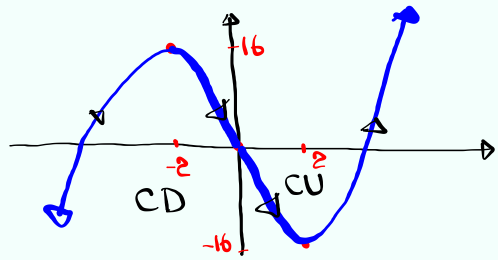
$f''(x) = 6x$

Domain of $f(x)$ $(-\infty, \infty)$

Points of interest for $x=-2$, $x=0$, and $x=2$

Sign Chart

x	$-\infty$	-2		0		2	∞
$f'(x)$	$+$	•	$-$		$-$	•	$+$
$f''(x)$	$-$		$-$	•	$+$		$+$
$f(x)$							



Apr 30-9:05 AM

Given $f(x) = 6\sqrt{x} - x\sqrt{x}$

$6\sqrt{2} - 2\sqrt{2} = 4\sqrt{2}$

1) Domain $[0, \infty)$
Even root
Radicand ≥ 0
$x \geq 0$

3) Y-Int.
$x = 0$
$y = 6\sqrt{0} - 0\sqrt{0} = 0$
$\to (0,0)$

2) x-Int
$y = 0$

$f(x) = 0$

$6\sqrt{x} - x\sqrt{x} = 0$

$\sqrt{x}(6-x) = 0$

$\to \sqrt{x} = 0 \to x = 0$

$6 - x = 0 \to x = 6$

$\to (0,0), (6,0)$

3) Find all points where $f'(x) = 0$ or undefined.

$f(x) = 6\sqrt{x} - x\sqrt{x}$

$f(x) = 6x^{1/2} - x \cdot x^{1/2}$

$f(x) = 6x^{1/2} - x^{3/2}$

$\to f'(x) = 6 \cdot \frac{1}{2} x^{-1/2} - \frac{3}{2} x^{\frac{1}{2}}$

$f'(x) = \frac{6}{2} x^{-1/2} - \frac{3}{2} x^{1/2}$

$f'(x) = \frac{3}{2} x^{-1/2}(2 - x)$

$f'(x) = \frac{3(2-x)}{2x^{1/2}}$

$f'(x) = \frac{3(2-x)}{2\sqrt{x}}$

$f'(x) = 0 \to 3(2-x) = 0$

$x = 2$

$(2, 4\sqrt{2})$

$f'(x)$ is undefined $\to 2\sqrt{x} = 0$

$x = 0 \to (0,0)$

Apr 30-9:13 AM

4) Find all points on the graph of $f(x)$ where $f''(x)=0$ or $f''(x)$ is undefined.

Recall $f'(x) = \frac{6}{2}x^{-1/2} - \frac{3}{2}x^{1/2}$

$$f''(x) = \frac{6}{2}\cdot\frac{-1}{2}x^{-3/2} - \frac{3}{2}\cdot\frac{1}{2}x^{-1/2}$$

$$= \frac{-6}{4}x^{-3/2} - \frac{3}{4}x^{-1/2}$$

$$= \frac{-3}{4}x^{-3/2}(2+x) = \frac{-3(x+2)}{4\sqrt{x^3}}$$

$f''(x)=0 \rightarrow -3(x+2)=0 \rightarrow x=-2$ Not matter Not in the domain

$f''(x)$ is undefined

$4\sqrt{x^3}=0$

$x=0$

$(0,0)$

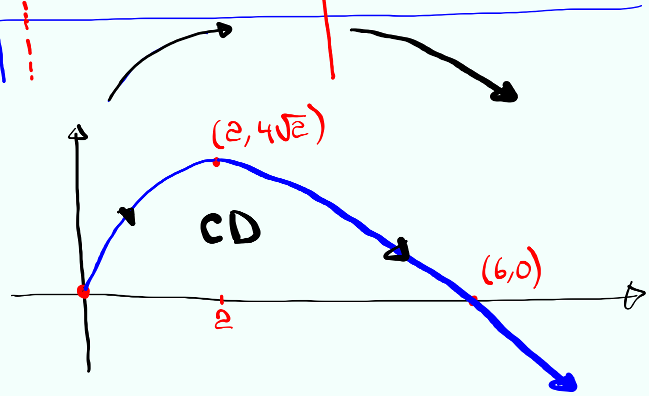
Apr 30-9:27 AM

$f'(x) = \frac{3(2-x)}{2\sqrt{x}}$ $\qquad$ $f''(x) = \frac{-3(x+2)}{4\sqrt{x^3}}$

Domain $[0,\infty)$

Points of interest $\rightarrow x=0,\ x=2$

x	0		2		∞
$f'(x)$	0	+	•	−	
$f''(x)$	0	−		−	
$f(x)$					



Apr 30-9:35 AM

Given $f(x) = \dfrac{4}{x^2+1}$

1) Domain $(-\infty, \infty)$

$x^2+1 \neq 0$

2) Y-Int $(0,4)$

3) x-Int.

$y=0 \rightarrow \dfrac{4}{x^2+1} = 0 \rightarrow 4 \neq 0$

None

4) Show $f(x)$ is an even function.

$f(-x) = \dfrac{4}{(-x)^2+1} = \dfrac{4}{x^2+1} = f(x)$

Since $f(-x) = f(x)$,

$f(x)$ is an even function.

Graph is symmetric w/t the Y-axis.

5) $\lim\limits_{x \to \infty} f(x) = \lim\limits_{x \to \infty} \dfrac{4}{x^2+1}$

$= 0$

6) $\lim\limits_{x \to -\infty} f(x) = 0$

Apr 30-9:42 AM

7) Find all points on the graph of $f(x)$ where $f'(x)=0$ or $f'(x)$ is undefined.

$f(x) = \dfrac{4}{x^2+1}$

$f(x) = 4(x^2+1)^{-1}$

$f'(x) = 4(-1)(x^2+1)^{-2} \cdot 2x$

$f'(x) = \dfrac{-8x}{(x^2+1)^2}$

$f'(x)=0$

$\rightarrow x=0$

$(0,4)$

$f'(x)$ is never undefined.

8) Find all points on the graph of $f(x)$ where $f''(x)=0$ or $f''(x)$ is undefined.

$f'(x) = \dfrac{-8x}{(x^2+1)^2}$

$f''(x) = -8 \cdot \dfrac{1(x^2+1)^2 - x \cdot 2(x^2+1) \cdot 2x}{[(x^2+1)^2]^2}$

$f''(x) = -8 \cdot \dfrac{\cancel{(x^2+1)}[x^2+1 - 4x^2]}{(x^2+1)^{\cancel{4}\,3}} = \dfrac{-8(1-3x^2)}{(x^2+1)^3}$

$f''(x)=0 \rightarrow 1-3x^2=0 \rightarrow x^2 = \frac{1}{3} \rightarrow x = \pm\frac{1}{\sqrt{3}}$

$f''(x)$ is never undefined.

$\left(\pm\frac{1}{\sqrt{3}}, 3\right)$

$f(x) = \dfrac{4}{x^2+1}$

$f\left(\pm\frac{1}{\sqrt{3}}\right) = \dfrac{4}{\frac{1}{3}+1} = 3$

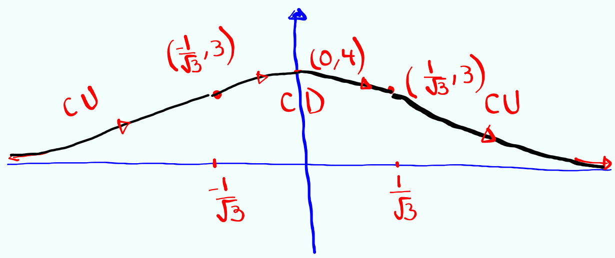
Apr 30-9:51 AM

$f'(x) = \dfrac{-8x}{(x^2+1)^2}$ $\qquad$ $f''(x) = \dfrac{8(3x^2-1)}{(x^2+1)^3}$

Domain $(-\infty, \infty)$

Points of interest $0, \pm\frac{1}{\sqrt{3}}$

x	$-\infty$	$-\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	∞
$f'(x)$	$+$	$+$		$-$	$-$
$f''(x)$	$+$	$-$		$-$	$+$
$f(x)$	↗	↗		↘	↘



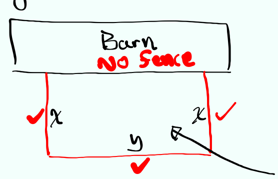
Range of $f(x)$ $\quad (0, 4]$

Apr 30-10:05 AM

I have 100 meters of fencing.

I want to build a rectangular dog walk next to a barn.

What dimensions of a rectangluar will give me a maximum area?



$2x + y = 100$

$y = 100 - 2x$

Area $= xy = x(100-2x)$

Area $\quad f(x) = 100x - 2x^2$

$f'(x) = 100 - 4x$

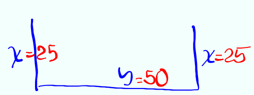
$f''(x) = -4 < 0$

Max.

$f'(x) = 0$

$100 - 4x = 0$

$\boxed{x = 25}$



Dimensions

25 m by 50 m.

Area

$25(50) = 1250\,m^2$

Apr 30-10:13 AM

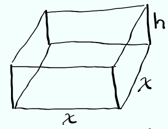
I have $\boxed{1200\ cm^2}$ of materials to make a box with square base, and open top.

$\to$ base + 4 Sides

$x^2 + 4xh = 1200$

$h = \frac{1200 - x^2}{4x}$

I want max. volume.



$V = LWH$

$V = x^2 h$

$V = \cancel{x^2}^{\,x}\left(\frac{1200 - x^2}{4\cancel{x}}\right)$

$V = \frac{1}{4}x(1200 - x^2) = \frac{1}{4}\left[1200x - x^3\right]$

$V'(x) = \frac{1}{4}\left[1200 - 3x^2\right]$

$V''(x) = \frac{1}{4}\left[-6x\right]$

$V'(x) = 0$

$\frac{1}{4}(1200 - 3x^2) = 0$

$\frac{1}{4} \neq 0$ $\quad 1200 - 3x^2 = 0$

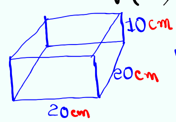
$400 - x^2 = 0$

$x = \pm 20$

only 20 works.

$V''(20) = -<0$

Max



$h = \frac{1200 - x^2}{4x}$

$= \frac{1200 - 20^2}{4(20)} = 10$

$V = 20 \cdot 20 \cdot 10 = 4000\ cm^3$

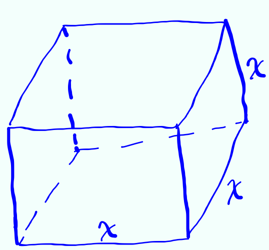
Google First - Derivative Test

Second - Derivative Test

Apr 30-10:21 AM

Volume of a cube is decreasing at the rate of $10\ cm^3/min$.

How fast its surface area changing when the edge is $20\ cm$?



$V = x^3$ $\qquad S = 6x^2$

$\frac{dV}{dt} = -10$ $\qquad \frac{dS}{dt}$ when $x = 20$.

$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$ $\qquad \frac{dS}{dt} = 12x\frac{dx}{dt}$

$-10 = 3 \cdot 20^2 \frac{dx}{dt}$ $\qquad \frac{dS}{dt} = \cancel{12} \cdot 20 \cdot \frac{-1}{\cancel{120}}$

$\frac{dx}{dt} = \frac{-10}{3 \cdot 400} = \frac{-1}{120}$ cm/min.

$= -2$ $cm^2/min.$

Apr 30-10:37 AM

Find the linear approximation to $\boxed{\sqrt{25-x^2}}$

$f(x)$

$\boxed{\text{near } 3}$.

$a=3$

$f(a) + f'(a)(x-a)$

$$\boxed{4 + \frac{-3}{4}(x-3)}$$

$f(x)=\sqrt{25-x^2}$

$f(3)=4$

Find $f'(x)$

$f(x)=(25-x^2)^{1/2}$

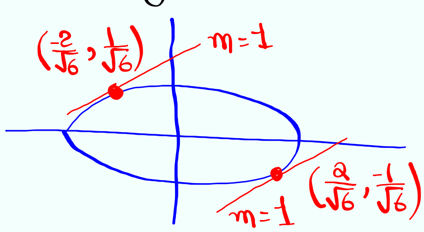
$f'(x)=\frac{1}{\cancel{2}}(25-x^2)^{-1/2}\cdot -\cancel{2}x$

$$f'(x)=\frac{-x}{\sqrt{25-x^2}}$$

$$f'(3)=\frac{-3}{\sqrt{25-3^2}}=\frac{-3}{4}$$

Apr 30-10:45 AM

Find the points on the graph of $x^2+2y^2=1$ has $\boxed{\text{Tan. line}}$ with $\boxed{\text{Slope } 1}$



$x^2+2y^2=1$

$2x + 4y\frac{dy}{dx}=0$

$\frac{dy}{dx}=\frac{-x}{2y}$

$1=\frac{-x}{2y}$

$2y=-x$

$-2y=x$

$x=-2\left(\pm\frac{1}{\sqrt6}\right)$

$x=\pm\frac{2}{\sqrt6}$

$x^2+2y^2=1$

$(-2y)^2+2y^2=1$

$4y^2+2y^2=1$

$6y^2=1$

$y^2=\frac{1}{6}$ $\quad y=\pm\frac{1}{\sqrt6}$

Apr 30-10:51 AM

Evaluate $\lim_{x \to \frac{\pi}{4}} \dfrac{\sin x - \frac{\sqrt{2}}{2}}{x - \frac{\pi}{4}} = \boxed{\dfrac{\sqrt{2}}{2}}$

Hint: $f'(a) = \lim_{x \to a} \dfrac{f(x) - f(a)}{x - a}$

$a = \frac{\pi}{4}$, $f(x) = \sin x$

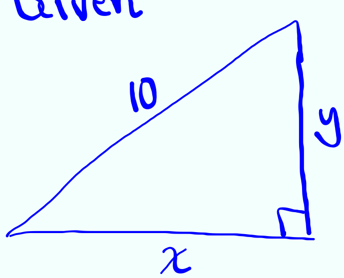
$f'(x) = \cos x$

$f'\left(\frac{\pi}{4}\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

Apr 30-10:58 AM

Class QZ 18

Given



x is decreasing at 4cm/min.

How fast is y changing when $y = 8$cm.

$\boxed{\text{Increasing}}$ or Decreasing?

$x^2 + y^2 = 10^2$

$x^2 + 8^2 = 10^2$

$x = 6$

$\to 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$(6)(-4) + 8\frac{dy}{dt} = 0 \to \boxed{\frac{dy}{dt}\ 3\text{cm/min}}$

Apr 30-11:01 AM